



# INTERNATIONAL JOURNAL OF ACADEMIC RESEARCH IN BUSINESS & SOCIAL SCIENCES



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To Link this Article: <http://dx.doi.org/10.6007/IJARBSS/v11-i4/9768>

DOI:10.6007/IJARBSS/v11-i4/9768

**Received:** 04 February 2021, **Revised:** 10 March 2021, **Accepted:** 28 March 2021

**Published Online:** 25 April 2021

**In-Text Citation:** (Azman & Maat, 2021)

**To Cite this Article:** Azman, N. A., & Maat, S. M. (2021). Integration of the History of Mathematics in Mathematics Education: A Systematic Literature Review. *International Journal of Academic Research in Business and Social Sciences*, 11(4), 1035–1057.

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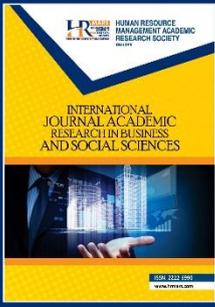
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Vol. 11, No. 4, 2021, Pg. 1035 - 1057

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# INTERNATIONAL JOURNAL OF ACADEMIC RESEARCH IN BUSINESS & SOCIAL SCIENCES



## Integration of the History of Mathematics in Mathematics Education: A Systematic Literature Review

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### Abstract

The history of mathematics (HoM) is important for mathematics to be enriched, in the evaluation of the development of the mathematics based on scholars and subjects. However, many students often have a bad impression of Mathematics. Therefore, a lot of alternatives have been used by Mathematics educators to change this sceptical view of Mathematics. One of the methods is by using the history in Mathematics in the teaching and learning process. Therefore, this study aims to identify Mathematics teachers' views towards the integration of the HoM and the implementation of HoM in teaching and learning in a systematic literature review context. Using the guidelines by Preferred Reporting Items for Systematic Review and Meta-Analyses (PRISMA) and the combination of keywords such as "integration of History of Mathematics", "using History of Mathematics", "implementing History of Mathematics" and "integrating History of Mathematics". 30 out of 1412 articles used in this study were chosen from two electronic databases which are ERIC, and Google Scholar without restricting the year of publication. The article selection was done based on a few criteria such that the variables in previous research were focused on the views and opinions of teachers about the usage of HoM and the usage of HoM in Mathematics education. Extraction of data was based on the name of the researcher, year, country, title, sample size, instrument used, field of study, methodology and analysis and findings. The findings reveal that Mathematics' teachers have positive views about the integration of HoM in teaching and learning. Teachers did not use HoM in class due to problems such as lack of knowledge and skill to integrate HoM, insufficient materials, resources and time and exclusion of HoM in the Mathematics syllabus and examination. To conclude, Mathematics teachers should consider integrating HoM in their teaching and choosing history as an alternative way to teach Mathematics. As for future works, Mendeley, Scopus and Web of Science are highly recommended to be databases of the study using should extensive related keywords.

**Keywords:** Integration, History of Mathematics, Systematic Literature Review.

### Introduction

Mathematics is a very important discipline in everyday life because it is a field that all people need to use and practice in everyday life (Bailey, 2017). However, Mathematics is

often labeled as a subject that is feared by most students not only at school level, but also among higher education students. Today, Mathematics anxiety is affecting most students because they think that Mathematics is difficult to learn because they cannot understand and master the concepts presented in this subject (Nachiappan et al., 2016). In addition, almost all of the concepts and elements learned in this subject are abstract and therefore students have difficulty in integrating them into their real lives (Bailey, 2017). In Malaysia, local research done by Hong Choon and Chandrasekaran (2015) found that every time a new mathematical symbol was introduced in Mathematics it had a negative impact on students' mathematics achievements. Despite this, some local Mathematics teachers came from non-mathematics background with a low level of content knowledge and this indirectly influenced their teaching performances as well as students' interest in learning Mathematics (Li et al., 2019). As a result, these factors cause students to have an idea that Mathematics is a hard, boring and unpleasant subject in school and that leaves them with negative attitudes towards it.

As such, various initiatives have been introduced by Mathematics teachers in order to help students who are experiencing phobias towards Mathematics to destroy and eliminate their fears and negative perceptions of this subject. One of the exciting initiatives that has begun to attract teachers' attention to enhance and diversify the teaching and learning of Mathematics in the classroom is by integrating the History of Mathematics (HoM) into Mathematics teaching. HoM is a scientific field which is known as one of the most crucial branches of Mathematical knowledge. This field describes the journey and development of Mathematics as well as the Mathematicians involved in the formulation of concepts as well as developing Mathematical formulas (Baki & Gürsoy, 2018).

Teachers choose to use HoM during their teaching sessions to develop affective and cognitive skills as well as to help students in understanding the nature of Mathematics. In addition, HoM is also used to ensure students' engagement in the classroom, gain their attention, arouse curiosity and help them to construct positive beliefs and attitudes towards Mathematics. HoM can also help students to understand the origins of a topic in Mathematics in addition to helping them to relate what they have learned in Mathematics to their everyday life (Başibüyük & Şahin, 2019). Bütüner and Baki (2020) stated that students' beliefs about Mathematics can be increased by applying HoM in Mathematics classes through fun, useful and interesting class activities. In addition, the usage of HoM also helps students to develop meaningful relationships between abstract mathematical ideas and practical applications in real-world contexts (Bailey, 2017). It is believed that the learning environment plays an important role in helping students to develop thinking skills in Mathematics (Hamidah et al., 2019). Therefore, the usage of HoM in teaching can be considered a brilliant step to help students in learning Mathematics, enrich teaching content and create an exciting learning environment. Moreover, the integration of HoM in the classroom has been widely supported by many researchers (Astin et al., 2016; Bidwell, 1993; Fauvel, 1991; Fried, 2001; Gulikers & Blom, 2001; Jankvist, 2009; Liu, 2003; Tzanakis et al., 2002). Several groups such as the National Council of Teachers of Mathematics (NCTM) and The Mathematical Association of America (MAA) have also proposed and recommended the integration of HoM in the classroom (Baki & Guven, 2009).

In a study conducted by Tzanakis et al. (2002), they highlighted several ways that teachers can integrate HoM in the teaching and learning process. These include (1) usage of historical quotes in teaching, (2) conducting research projects involving the use of historical sources, (3) usage of original sources for a topic, (4) incorporating HoM elements in practising

of mathematics questions (5) discussing problems arising from the discovery of Mathematics based on its history, (6) use of mechanical materials, (7) conducting experimental-based Mathematical activities, (8) presenting HoM in the form of games, (9) using film, video or other visual materials, (10) emphasising external experiences and (11) encouraging the search for the origin of a mathematical concept through the internet.

In addition, most studies have found that the usage of HoM in teaching Mathematics has many positive effects on teachers, students and the process of teaching and learning Mathematics itself (Hickman & Kapadia, 1983; Philippou & Christou, 1998; Marshall & Rich, 2000; Liu, 2003; Charalambous, Panaoura, & Philippou, 2009). HoM is also seen as a medium that can serve as a guide for developing mathematical instruction designs that include a variety of aspects such as teaching strategies, problem solving and interesting topics that can be used for classroom discussion (Galante, 2014). Therefore, this systematic literature review aims to review articles on integration of HoM in Mathematics education. The objectives of this study are as follows:

- To identify Mathematics teachers' views on the integration of HoM in Mathematics teaching and learning sessions.
- To investigate the implementation of HoM in teaching and learning Mathematics.

## **Methodology of Research**

### *General Background of the Research*

This study uses a systematic literature review method. According to Cooper (2003), systematic review is a method for determining, assessing critically and deciphering research discoveries identified with research questions, topics or the significance of such phenomena. Therefore, this is the best method to study related articles on the integration of HoM. Thirty articles were involved in this study on Mathematics teachers' views and opinions about the integration of HoM in the classroom including the practice of the integration of HoM in teaching and learning processes as well as the benefits of it. A qualitative analysis method has been used in this study by conducting content analysis to analyse the results for this study. The guidelines introduced by Moher, Liberati, and Tetzlaff, (2009) in Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) is followed in order to handle and report the outcomes of this study.

### *Selection Criteria*

This study only considered articles generated from article writing on the integration of HoM in Mathematics education without restricting the year of publication. Sources of writing such as blogging and unpublished reports were excluded in this study because these types of sources do not go through the peer review process. Thus, the credibility and objectivity of the writing should be taken into account (Liyanagunawardena et al. 2013). The selected articles are either qualitative or quantitative and involve only open-access articles to ease the downloading process for the full-text articles. This study also involved articles which are written in English only.

### *Searching Strategy*

The articles involved in this study were chosen by focusing on the integration of HoM in Mathematics education and published at any time. Two electronic databases, the Education Resources Information Center (ERIC) and Google Scholar, were used to find the related articles. These databases were chosen because Google Scholar is a database that has

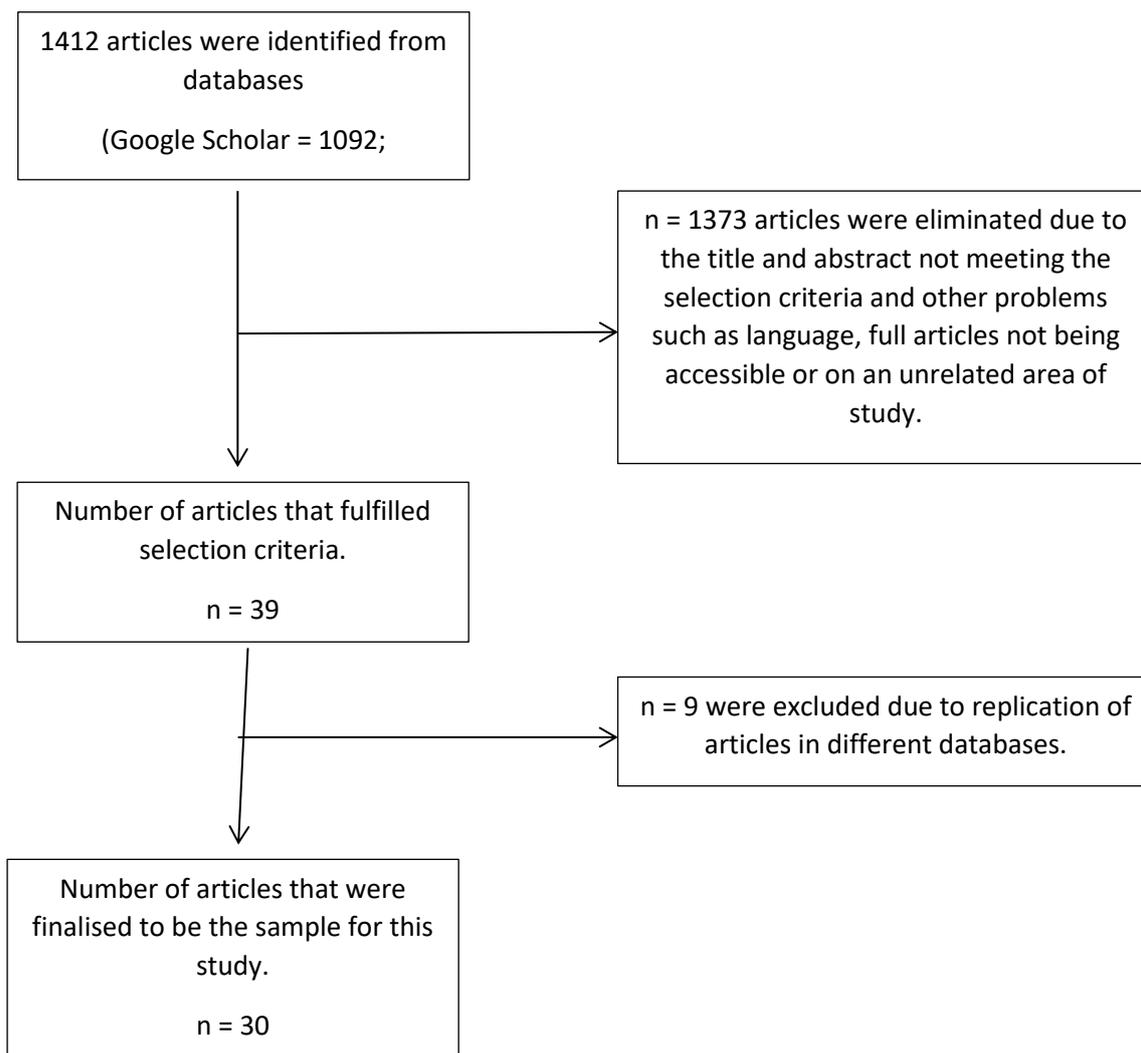
accumulated articles from different databases while ERIC is a database that can be used to find educational related articles. The combination of keywords that was used to conduct searches in the databases listed was: “integration of History of Mathematics”, “using History of Mathematics”, “implementing History of Mathematics” and “integrating History of Mathematics”. The articles and abstracts went through preliminary checks before finding the full text for the articles that met the criteria. Final selection of articles was determined by selecting only the articles that satisfied the needed criteria.

#### *Data Extraction*

For the preliminary check, the researcher went through the title of the article and abstract to select suitable articles. Articles that did not satisfy the criteria needed were excluded. Then, the researcher downloaded the full text of the remaining articles and read all of them to determine which articles needed to be included or excluded in this study. The researcher then constructed a table for data extraction including the name of the researcher(s), publication year, publication country, title, sample size, instrument used, variable, methodology and analysis and lastly the research findings.

#### **Results of Research**

This paper aims to study articles relating to the integration of the History of Mathematics (HoM). This systematic survey explores Mathematics teachers’ views about the integration of HoM in Mathematics education and the usage of HoM in Mathematics education. The flow of the article selection is shown in Figure 1.

**Figure 1: The flowchart of article selection**

Initially, 1412 articles were identified by the search protocol. Overall, 1373 were then eliminated after going through an inspection by title and abstract. Most of the articles were eliminated due to the different areas of study related to HoM and different characteristics of the sample studied. Next, 39 articles were left. From the articles left, 9 were eliminated due to replication of articles in different databases. Finally, 30 articles were included in this systematic review study.

Table 1 below shows a summary of the articles involved in this systematic survey. From 30 articles that matched the selection criteria for this survey, 12 were done in Turkey, 5 in the United States, 2 in Indonesia, 2 in Singapore, 1 respectively in Ethiopia, Greece, Germany, Taiwan, Denmark, Serbia, South Africa, China and Australia. Fifteen of the selected articles studied were on the views of Mathematics' teacher about HoM while 19 articles studied were on the integration of HoM in Mathematics education. The methods used to study the opinions about HoM were: test, questionnaire, interview, survey and observation and also reflective writing. Meanwhile, the methods used to study the practice of HoM were: knowledge test, written opinion form, reflective journals, constant comparative method and case study.

**Table 1. Summary of selected articles**

No	Researcher (s)	Country	Title	Population	Instrument	Field of study	Methodology/ Analysis	Findings
1	Fraser & Koop (1978)	Australia	Teachers' opinions about some teaching material involving the history of mathematics	39 mathematics teachers	Questionnaires	Views about HoM	T-test	Teachers' have positive impressions about using HoM materials but they did not consider using it in teaching because of a lack of time and skills
2	Tzanakis et al. (2002)	Greece	Integrating history of mathematics in the classroom: an analytic survey	-	Analytic survey	Integration of HoM	Content analysis	Ways to integrate HoM in class: give direct information, use historical-inspired teaching strategy, spread awareness in social and cultural contexts of Mathematics. Examples of tools to integrate HoM in class: historical snippets, history-based projects, primary sources, worksheets, historical packages, discussion on historical issues and arguments, historical problems, mechanical instruments, experiential activities, plays, visual tools, outdoor learning and internet sources.
3	Liu (2003)	Taiwan	Do Teachers Need to Incorporate the History of	-	Content analysis	Integration of HoM	Literature review	Reasons to integrate HoM in teaching: develop positive attitudes and increase motivation towards learning Mathematics, help to explain difficulties faced by students

			Mathematics in their Teaching?					in problem solving, trigger students' mathematical thinking and explain human side of mathematical knowledge.
4	Ho (2008)	Singapore	Using history of mathematics in the teaching and learning of mathematics in Singapore	102 students	1. Record of the teaching and learning process 2. Student survey	Integration of HoM	1. Case study 2. Qualitative analysis	Benefits of integrating HoM: better understanding, creating different learning environment and developing positive attitudes towards teaching and learning Mathematics. Limitation: lack of teachers training, time and rubrics Risks of using HoM: cannot make a relevant connection between HoM and the topic, time issues and wrong interpretation
5	Jankvist (2009)	Denmark	A categorisation of the whys and hows of using history in mathematics education.	-	Content analysis	Integration of HoM	Literature review	Usage of HoM: teaching and learning tools, as a goal in development of Mathematics as a discipline. How to use HoM in education: Illumination approach, Modules approach and History-based approach
6	Burns (2010)	United States	Pre-Service Teachers' Exposure to Using the History of Mathematics to Enhance Their	32 pre-service teachers	Questionnaire, open-ended question and final reflection paper	Views about HoM	Paired sample t-test Grounded theory approach	Pre-service teachers have positive views about integrating the HoM after being exposed in terms of how to use HoM.

			Teaching of High School Mathematics					
7	Bolinger-Horton & Panasuk (2011)	United States	Raising Awareness of the History of Mathematics in High School Curriculum	367 Mathematics teachers	Survey	Integration of HoM	Independent sample T-test analysis	Teachers who have a fallibilist perspective towards the nature of Mathematics are more likely to include HoM in their lessons compared to teachers who have an absolutist perspective towards the nature of Mathematics. Teachers who have an absolutist perspective have a fixed procedure in their teaching process and are less interested in creating a new learning environment while teachers with a fallibilist perspective are flexible to change their teaching style.
8	Panasuk & Bolinger-Horton (2011)	United States	Integrating History of Mathematics into Curriculum: What are the Chances and Constraints?	367 Mathematics teachers	Online comprehensive survey	1. Views about HoM 2. Integration of HoM	Factor analysis	Most of the teachers have good perceptions of HoM. They believed that HoM has its value and place in the school curriculum and would help to enhance students' motivation and interest in learning Mathematics. However, some of the teachers viewed HoM as burdening. Factors that affect teachers' decision in applying HoM are their beliefs about the importance of HoM, benefits of HoM and values of HoM.

9	Alpaslan & Haser (2012)	Turkey	"History Of Mathematics" Course For Pre-Service Mathematics Teachers: A Case Study	24 pre-service mathematics teachers	Observation of the course setting, interviews	Views about HoM	Descriptive analysis	Pre-service teachers have positive attitudes and beliefs about the usage of HoM but did not consider that learning about HoM is so important.
10	Ozdemir, Goktepe, & Kepceoglu (2012)	Turkey	Using mathematics history to strengthen geometric proof skills	15 students from 11 <sup>th</sup> grade	Questionnaire	Integration of HoM	Multiple case study	Using HoM helped to strengthen students' geometric proof skills besides attracting their interest to learn the topic.
11	Goktepe & Ozdemir (2013)	Turkey	An example of using history of mathematics in classes	21 students in private primary school	1. Worksheets 2. Interview	Integration of HoM	Multiple case study	Integration of HoM in classroom teaching attracts students' interests and helps them to discover different methods to solve mathematical problems.
12	Panasuk & Horton (2013)	United States	Integrating History of Mathematics into the Classroom: Was Aristotle Wrong?	367 Mathematics teachers	On-line comprehensive survey	Integration of HoM	Descriptive analysis	Factors influencing usage of HOM: teacher knowledge, resources, high stakes testing and enjoyment in using HOM.
13	Alpaslan, İşiksal,	Turkey	Pre-service Mathematics Teachers' Knowledge of	1593 pre-service Mathematics teachers	1. Knowledge of History of Mathematics Test	Views about HoM	1. Cross-sectional survey	Pre-service teachers have moderate knowledge of HoM and positive attitudes and beliefs about using HoM.

	& Haser (2014)		History of Mathematics and Their Attitudes and Beliefs Towards Using History of Mathematics in Mathematics Education		2. Attitudes and Beliefs about the Use of History of Mathematics in Mathematics Education Questionnaire		research design 2. Descriptive statistics 3. Two-way Analysis of Variance	
14	Dejic & Mihajlovic (2014)	Serbia	History of Mathematics and Teaching Mathematics	112 teachers	Survey	1. Views about HoM 2. Integration of HoM	Quantitative analysis	Teachers have positive perceptions about HoM but they did not integrate HoM in their teaching due to lack of resources, methodology and time.
15	Galant (2014)	United States	The Use of the History of Mathematics in the Teaching of Preservice Mathematics Teachers	9 pre-service teachers	1. Reflective journals written by participants 2. Constant comparative method 3. Case study	Views about HoM	1. Grounded theory approach 2. Qualitative analysis	Pre-service teachers believed usage of HoM in teaching is beneficial in the context of content knowledge and pedagogical content knowledge.
16	Weldeana & Abraham (2014)	Ethiopia	The effect of an historical perspective on prospective teachers' beliefs in learning mathematics	63 pre-service teachers	Questionnaire Reflective writing	Views about HoM	T-test Kitwood's method of analysing accounts	History-based intervention provides a positive impact in shifting prospective teachers' beliefs to progressive beliefs.

17	Bütüne r (2015)	Turkey	Impact of Using History of Mathematics on Students' Mathematics Attitude: A Meta-Analysis Study	6 studies	Content analysis	Integratio n of HoM	Meta-analysis method	Using HoM in Mathematics education develops students' positive attitude to learning Mathematics.
18	Fadleml ula (2015)	Turkey	Pre-Service Teachers' Point Of Views About Learning History of Mathematics: A Case Study In Turkey	120 pre-service teachers	Open-ended questionnaire	Views about HoM	Descriptive statistics	Pre-service teachers had positive attitudes towards learning HoM. They believed HoM is beneficial for both personal and pedagogical aspects. However, they had limited ideas on how to implement HoM in teaching.
19	Lim & Chapm an (2015)	Singapo re	Effects of using history as a tool to teach mathematics on students' attitudes, anxiety, motivation and achievement in grade 11 classrooms	103 students	Tests	Integratio n of HoM	1. Quasi-experiment 2. Qualitative feedback	Implementing HoM in the class provides both immediate short-term and long-term effects on students' achievement in Mathematics. However, implementation of HoM only has a short-term positive effects on students' attitudes, anxiety and motivation towards Mathematics.
20	Tol (2015)	Turkey	Teachers' Views Related To Teaching Of Mathematics	20 Mathematic s teachers	Open-ended questions	Views about HoM	Descriptive statistics and content analysis	Teaching using HoM is more effective to ensure permanent learning. Teachers agreed with the idea of

			Course Subjects With Their Historical Developments					using HoM in class but rarely use it due to not having sufficient material.
21	Bütüne r(2017)	Turkey	Secondary School Mathematics Teachers' Knowledge Levels and Use of History of Mathematics	32 Mathematics teachers	1. 11-item short-answer HoM knowledge level test 2. 5-item written opinion form	Integration of HoM	Descriptive analysis	Most teachers did not use HoM in their classes because they feel incompetent and had low HoM knowledge levels.
22	Baki & Gürsoy (2018)	Turkey	Does Using History of Mathematics Make Sense? The Views of Teacher Candidates	6 teacher candidates	Semi-structured interview	Views about HoM	Qualitative analysis	Teacher candidates believe that using HoM in the instruction of Mathematics may benefit both teaching and learning purposes. However, they have an issue practicing this because of lack of resources and time
23	Şişman (2018)	Turkey	History of Mathematics in the Turkish Middle School Mathematics Curriculum and Textbooks	Turkish middle school mathematics curriculum (5th-8th grades) and textbooks	Curriculum and textbooks	Integration of HoM	Document analysis	Most of the HoM elements found in the Mathematics textbooks were placed in the introduction part of the topic. HoM also was excluded from the content and assessment aspects of the curriculum. It can be concluded that HoM is not being integrated widely in the school Mathematics curriculum.

24	Başibüyük & Şahin (2019)	Turkey	Mathematics Teachers' Opinion About The History of Mathematics	21 Mathematics teachers	Structured interview form	1. Views about HoM 2. Integration of HoM	1. Qualitative method of case study 2. Content analysis	Teachers felt incompetent to utilise HoM in teaching because of a lack of knowledge in HoM, lack of resources and its exclusion in the syllabus and examination. They commonly use HoM by telling the biographies of Mathematicians instead of focusing on Mathematics information.
25	Buchholtz & Schorcht (2019)	Germany	Different Facets of Pre-Service Teachers' Beliefs On The History of Mathematics	141 pre-service teachers	Questionnaires	1. Views about HoM 2. Integration of HoM	1. Descriptive analysis 2. CFA	Overall, majority of pre-service teachers support and have positive beliefs about the usage of HoM in the classroom. Factors that affect their beliefs are timing, relevance and complexity of using HoM in teaching.
26	Kusumawati & Fachrudin (2019)	Indonesia	Analisis Sikap dan Keyakinan Calon Guru di Indonesia terhadap Pemanfaatan Sejarah Matematika dalam Pembelajaran Matematika	349 teacher candidates	Attitude and Belief Questionnaires	Views about HoM	Quantitative descriptive analysis	Attitudes and beliefs on the usage of HoM of teacher candidates are at a moderate level. Besides, the self-efficacy scores in using HoM in teaching and learning are still low.
27	Bütüner & Baki (2020)	Turkey	The Use of History of Mathematics in the Mathematics	24 8 <sup>th</sup> grade students	1. Written opinion form 2. Semi-structured interviews	Integration of HoM	Action Study	Students found Mathematics is fun and interesting after being involved in activities related to the HoM in classroom. HoM also helped to

			Classroom: An Action Study					strengthen their beliefs about Mathematics.
28	Kapofu & Kapofu (2020)	South Africa	“This Maths is better than that Maths” – Exploring Learner Perceptions on the Integration of History of Mathematics in Teaching the Theorem of Pythagoras: A Case Study	30 grade 11 <sup>th</sup> students	1. Group interviews 2. Journal entries	Integration of HoM	Case study	Integration of HoM in Mathematics classrooms develops students’ positive perceptions about Mathematics and increases their motivation and readiness to learn Mathematics.
29	Kusumawati et al. (2020)	Indonesia	History of Mathematics for Teaching Mathematics: The Case of Indonesian Prospective Teachers' Beliefs and Attitudes	305 prospective teachers	Questionnaires of Attitude and Belief in using HoM to teach Mathematics	Views about HoM	Independent sample t-test	Prospective teachers had positive attitudes and beliefs about using HoM to teach Mathematics. Those who had taken the HoM course had a higher score compared to those who had not taken the course before. However, both groups showed low scores in the questionnaire item on self-efficacy beliefs.
30	Li & He (2020)	China	Collision Between Mathematics History and High School Teaching	-	Content analysis	Integration of HoM	Qualitative analysis	HoM is beneficial for students by developing a new learning environment, increasing students’ enthusiasm to learn Mathematics, get knowledge about the origin of Mathematics, encourage students to

			From HPM Perspective					think and construct ideas by themselves, explore the beauty in learning Mathematics and allowing them to fall in love with Mathematics.
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## Discussion

### Mathematics Teachers' Views about the Integration of HoM

From Table 1, it can be summarized that most of the teachers have positive views about the integration of the History of Mathematics (HoM) (Alpaslan & Haser, 2012; Alpaslan, İşiksal, & Haser, 2014; Baki & Gürsoy, 2018; Burns, 2010; Dejjic & Mihajlovic, 2014; Fadlelmula, 2015; Fraser & Koop, 1978; Galante, 2014; Kusumawati et al., 2020; Panasuk & Bolinger Horton, 2011; Buchholtz & Schorcht, 2019; Tol, 2015; Weldeana & Abraham, 2014). Three of the articles revealed that most of the teachers agreed that practicing the integration of HoM in their teaching would be beneficial for them as well as the students (Baki & Gürsoy, 2018; Fadlelmula, 2015; Galante, 2014). Panasuk & Bolinger-Horton (2011) believed that HoM has its place and values in the school Mathematics curriculum. Two of the articles stated that integrating HoM in the teaching and learning process could help to ensure effective and permanent learning of the students, enhance students' motivation and increase students' interest in learning Mathematics (Panasuk & Bolinger-Horton, 2011; Tol, 2015). Content knowledge and pedagogical content knowledge possessed by a teacher can be strengthened by applying HoM in teaching. Including HoM in teaching is also seen as one of the best alternative ways or useful strategies to deliver Mathematical knowledge effectively to the students in school (Galante, 2014). One of the articles revealed that teachers who had taken an HoM course in their study before had higher attitude and belief scores towards using HoM in teaching compared to ones who had not taken a course before (Kusumawati et al., 2020).

Kusumawati et al. (2020) in their studies, revealed that teachers' self-efficacy beliefs towards using HoM in their teaching are still low. Two of the studies stated that the teachers did not consider applying the integration of HoM in their teaching although they had positive views about the integration of HoM in teaching (Alpaslan & Haser, 2012; Fraser & Koop, 1978). Alpaslan and Haser (2012) in their studies, stated that teachers did not consider using HoM in teaching because they felt that using HoM in Mathematics education is not so important. In addition, using HoM materials in class might lead to wasting time if it is not fully prepared because they require the teachers to possess specific skills to apply HoM materials in their teaching (Fraser & Koop, 1978).

Despite having positive views about HoM, some of the studies revealed that many teachers encountered problems in applying HoM in class (Alpaslan et al., 2014; Baki & Gürsoy, 2018; Başibüyük & Şahin, 2019; Bütüner, 2017; Tol et al., 2015). Three of the articles mentioned knowledge of HoM as the factor in integrating HoM in teaching (Alpaslan & Haser, 2012; Başibüyük & Şahin, 2019; Bütüner 2017). Bütüner (2017); Başibüyük and Şahin (2019) in their studies, stated that teachers did not utilise HoM in teaching due to a lack of knowledge on HoM. Bütüner (2017) stated that teachers in general had a low level of knowledge in HoM and those who did not practice HoM had a lower level of knowledge in HoM compared to ones who applied it in teaching. Alpaslan and Haser (2012) revealed that pre-service teachers had moderate knowledge of HoM and their knowledge of HoM increased as their teaching year increased. It was also revealed that the knowledge scores on HoM of male teachers were slightly higher than female teachers.

Another factor that affects teachers' views of applying HoM in teaching is a lack of resources and materials (Baki & Gürsoy, 2018; Başibüyük & Şahin, 2019; Dejjic & Mihajlovic, 2014; Tol, 2015). Buchholtz & Schorcht (2019) revealed that some pre-service teachers think that HoM is too complex for students to learn. Besides that, three articles mentioned a lack of time as the factor (Baki & Gürsoy, 2018; Dejjic & Mihajlovic, 2014; Schorcht & Buchholtz, 2019). One article stated that teachers have no idea where and how to integrate HoM in

teaching (Fadlelmula, 2015) and three articles emphasised its exclusion in the syllabus and examination as a factor for why teachers did not apply HoM in class (Başibüyük & Şahin, 2019; Bütüner, 2017; Schorcht & Buchholtz, 2019).

### **Implementation of HoM in Teaching and Learning Mathematics**

Based on Table 1, four of the articles had done the research on the integration of HoM in Mathematics education. Two of the articles revealed that most of the teachers did not implement HoM in their teaching because they felt incompetent to do so due to problems such as having a low level of knowledge in HoM, a lack of resources and materials, a lack of time and exclusion of HoM from the Mathematics syllabus and examination (Başibüyük & Şahin, 2019; Dejjic & Mihajlovic, 2014; Bütüner, 2017). One of the articles' findings stated that instead of focusing on Mathematical concepts and information, teachers commonly practice HoM in their teaching by telling the biographies and life stories of Mathematicians (Başibüyük & Şahin, 2019). Buchholtz & Schorcht (2019) in their study also revealed that some pre-service teachers think that HoM is too complex for students to learn. Thus, they rejected the usage of HoM in their teaching.

Besides that, one article discussed the factors influencing the integration of HoM in teaching and learning Mathematics. Factors that influenced teachers' decisions in terms of integrating HoM into their teaching are their beliefs about the importance of HoM, benefits of HoM and values of HoM in the Mathematics curriculum (Panasuk & Horton, 2013). Panasuk and Horton (2013) listed the factors that might become problems for a teacher in applying HoM in their teaching: (1) level of knowledge of HoM; (2) materials and resources; (3) high stakes testing and (4) enjoyment of teaching and learning HoM. Te Bolinger-Horton & Panasuk (2011) teachers who have a fallibilist perspective about the nature of Mathematics are more likely to integrate HoM in their lessons while teachers who have an absolutist perspective about the nature of Mathematics are less likely to integrate HoM in teaching. Teachers who have an absolutist perspective have a rigid procedure in their teaching process and are less interested in creating new learning opportunities while teachers with a fallibilist perspective are open to changes and development in their teaching style.

Liu (2003) listed five reasons why teachers need to implement HoM in teaching. The first reason is HoM helps to increase students' motivation besides evolving a positive attitude towards learning Mathematics. This is supported by the finding from meta-analysis research done by Bütüner (2015) that stated that using HoM in Mathematics education develops students' positive attitude to learning Mathematics. Integration of HoM in the Mathematics classroom develops students' positive perceptions about Mathematics as well as increasing their motivation and readiness to learn Mathematics (Kapofu & Kapofu, 2020). Next, HoM also helps to explain what students find difficult now in solving Mathematical problems by referring to the obstacles faced in the history of the development of Mathematics. Besides, HoM helps to develop students' mathematical thinking, reveals the human side of mathematical knowledge and can act as a guide to Mathematics teachers for teaching.

Jankvist (2009) in his research, listed why we need to use HoM in Mathematics education and how to use HoM in Mathematics education. HoM is highly recommended in teaching because: (1) history acts as a tool to assist actual teaching and learning of Mathematics in the classroom and (2) history acts as a goal in the evolution of Mathematics as a subject. HoM can be applied in the teaching and learning process by using three approaches. First, there is the illumination approach. In this approach, historical information is used in teaching. Second, there is the modules approach. In the modules approach, teachers

can use history-based materials when teaching a small topic. Third is the history-based approach where the idea of teaching is directly inspired by or based on HoM.

HoM may be implemented in Mathematics education in three ways, which are learning history by giving direct information on history, learning Mathematical topics by using a historical-inspired teaching strategy, and developing a deepened awareness of the social and cultural contexts of Mathematics. These are a few examples of tools and ideas on how to integrate HoM in class: (1) historical snippets, (2) history-based projects, (3) primary sources, (4) worksheets, (5) historical packages, (6) discussion on historical issues and arguments, (7) history-related problems, (8) mechanical tools, (9) experiential learning, (10) role playing and games, (11) visual tools, (12) outdoor learning and (13) internet sources (Tzanakis et al., 2002).

Ho (2008) listed the potential, limits and risks of implementing HoM in teaching and learning Mathematics. The possible potentialities of using HoM in the classroom are: (i) understanding the topic much better, (ii) presenting different learning situation and (iii) developing positive attitudes of the teachers and students towards teaching and learning Mathematics. However, teachers will face a few limitations in applying HoM in their teaching: (i) a lack of training, (ii) a lack of time and (iii) a lack of rubrics related to HoM. Other than that, implementation of HoM in teaching and learning might involve a few risks such as: (i) teachers being unable to make a relevant connection between HoM and the topic, (ii) it may lead to wasting time if the lesson is not planned carefully, and (iii) students are not familiar with the cultural differences from the past and might wrongly interpret and understand the word 'history' in HoM.

It is found that implementing activities related to HoM in teaching and learning attract students' interests. Li & He (2020) found that HoM is beneficial for students by developing a new learning environment, increasing students' enthusiasm to learn Mathematics, knowledge on the origin of Mathematics, encouraging students to think and construct ideas by themselves, exploring the beauty in learning Mathematics and letting them fall in love with Mathematics. Students also have the opportunity to discover different methods to solve mathematical problems (Goktepe & Ozdemir, 2013). In addition, practising activities related to HoM in class can develop a fun and interesting learning experience for the students and deepen their beliefs towards Mathematics (Bütüner & Baki, 2020). This is supported by Ozdemir et al. (2012) in their study who have found that HoM helps to strengthen students' geometric proof skills besides attracting their interest in terms of learning the topic. Implementing HoM in the class also has both short-term and long-term effects on students' achievement in Mathematics. Despite this, the implementation of HoM only has short-term positive effects on students' attitudes, anxiety and motivation towards Mathematics (Lim & Chapman, 2015).

HoM has been integrated in Indonesian Mathematics textbook since 2013 in the form of historical snippets. It is found that this helps students to understand the concepts in Mathematics more deeply. HoM also increases students' motivation and encourages active involvement in the Mathematics' classroom (Ekawati et al., 2018). In Turkey, HoM has also been integrated in Mathematics textbooks. However, most of the HoM elements were only placed in the introduction part of the topics. HoM was also excluded from the content and assessment aspects of the curriculum. It can be concluded that HoM is still not acknowledged as beneficial and not integrated widely in school Mathematics curricula (Şişman, 2018).

## Conclusions

In conclusion, these systematic review findings have revealed that most of the teachers have positive views, attitudes, and beliefs about using the History of Mathematics (HoM) in teaching and learning Mathematics. Most of them believe that using HoM in Mathematics education is beneficial to them as well as the students. However, the implementation of HoM in teaching is still low due to several problems that need to be addressed by teachers, such as the level of knowledge of HoM, lack of resources and materials, lack of time and exclusion from the syllabus and examination. As for future works, Mendeley, Scopus and Web of Science are highly recommended to be databases of the study using should extensive related keywords.

## Acknowledgement

This research was funded by the grant of Faculty Education, Universiti Kebangsaan Malaysia GG-2019-065.

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